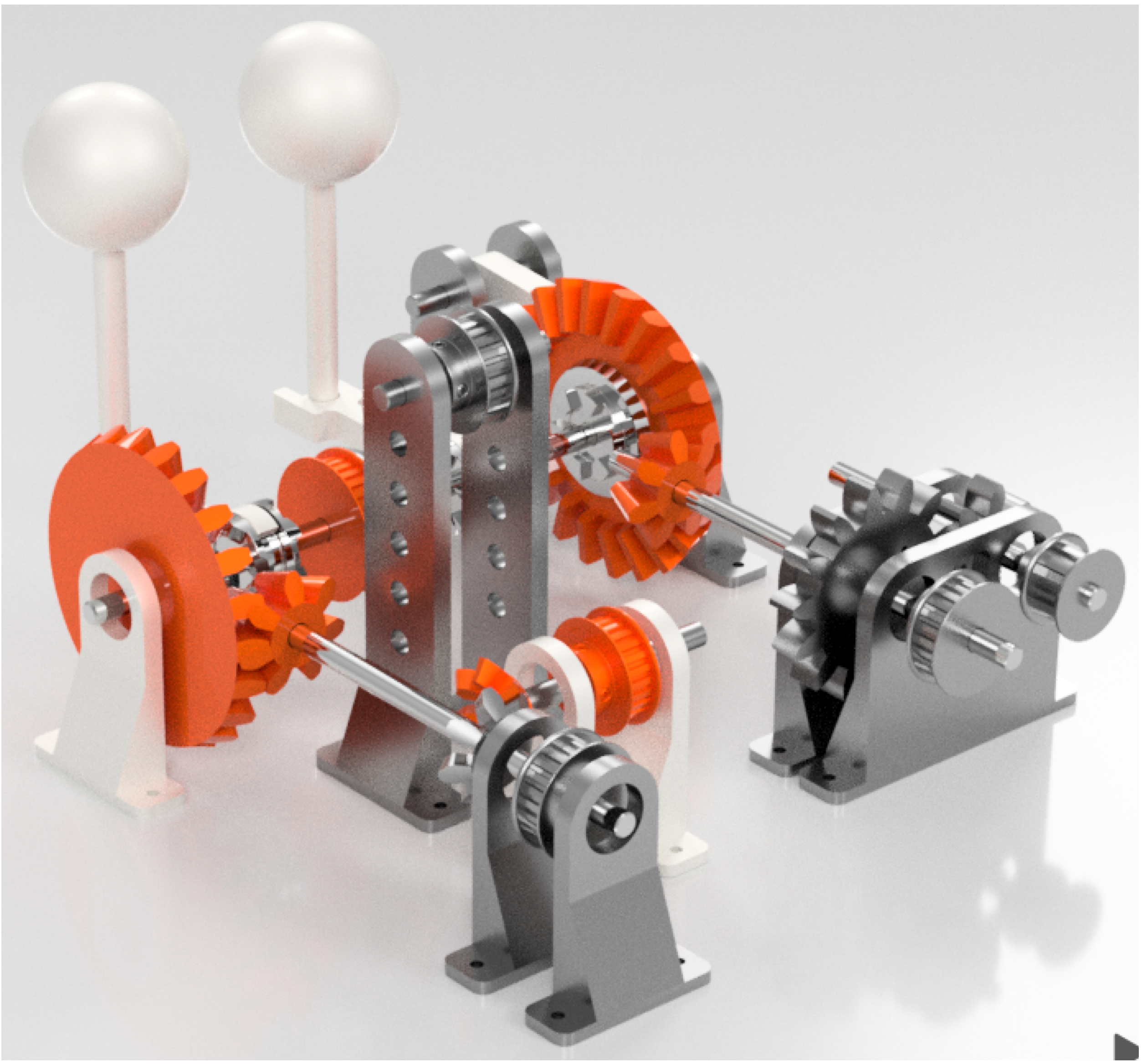
ME 371 Design Project

Final Report



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Project Overview

Our team has been tasked with designing a transmission capable of lifting a 4 kg weight at three different speeds: high, low, and reverse. The transmission must also be capable of being idle. The key components of our design include a gear-belt timing pulley to control the high-speed mechanism and two bevel gear trains to control the reverse and low-speed mechanisms. Our lift speed and gear ratio for each mechanism can be seen in Table 1 below. A 2:1 spur gear train connects the input shaft to our transmission, dividing our input speed by half to reduce intermediate gear ratios. Another gear-belt allows the reverse and low-speed mechanisms to spin at the same speed. The beveled gear train that controls the reverse direction is flipped 180 degrees on the output shaft to switch the direction of rotation. To allow the transmission to go idle, we have custom designed a ratchet gear and pawl that attaches to the reverse mechanism and will be triggered once the shifter is put in the neutral position. Our custom-designed shifters and dog clutches allow a user to smoothly switch between speeds.

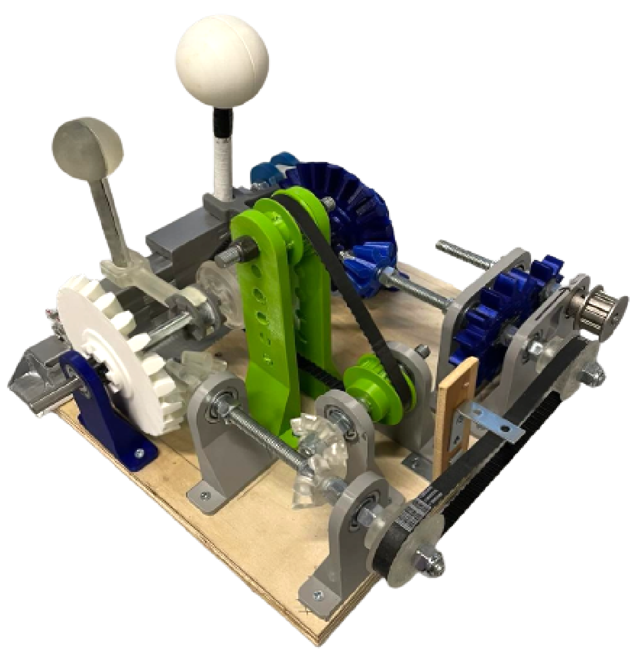

Figure 1. Final Benchmark Transmission Design

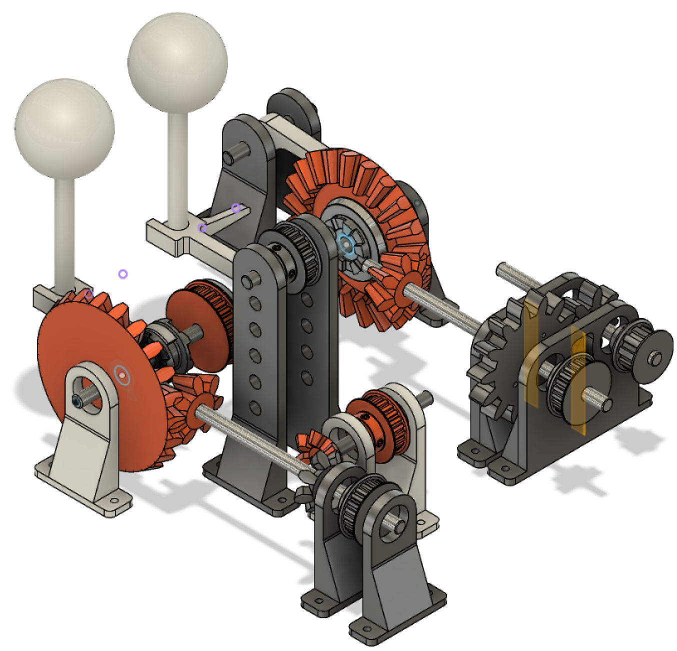

Figure 2. Fully Assembled CAD Model of Transmission Design

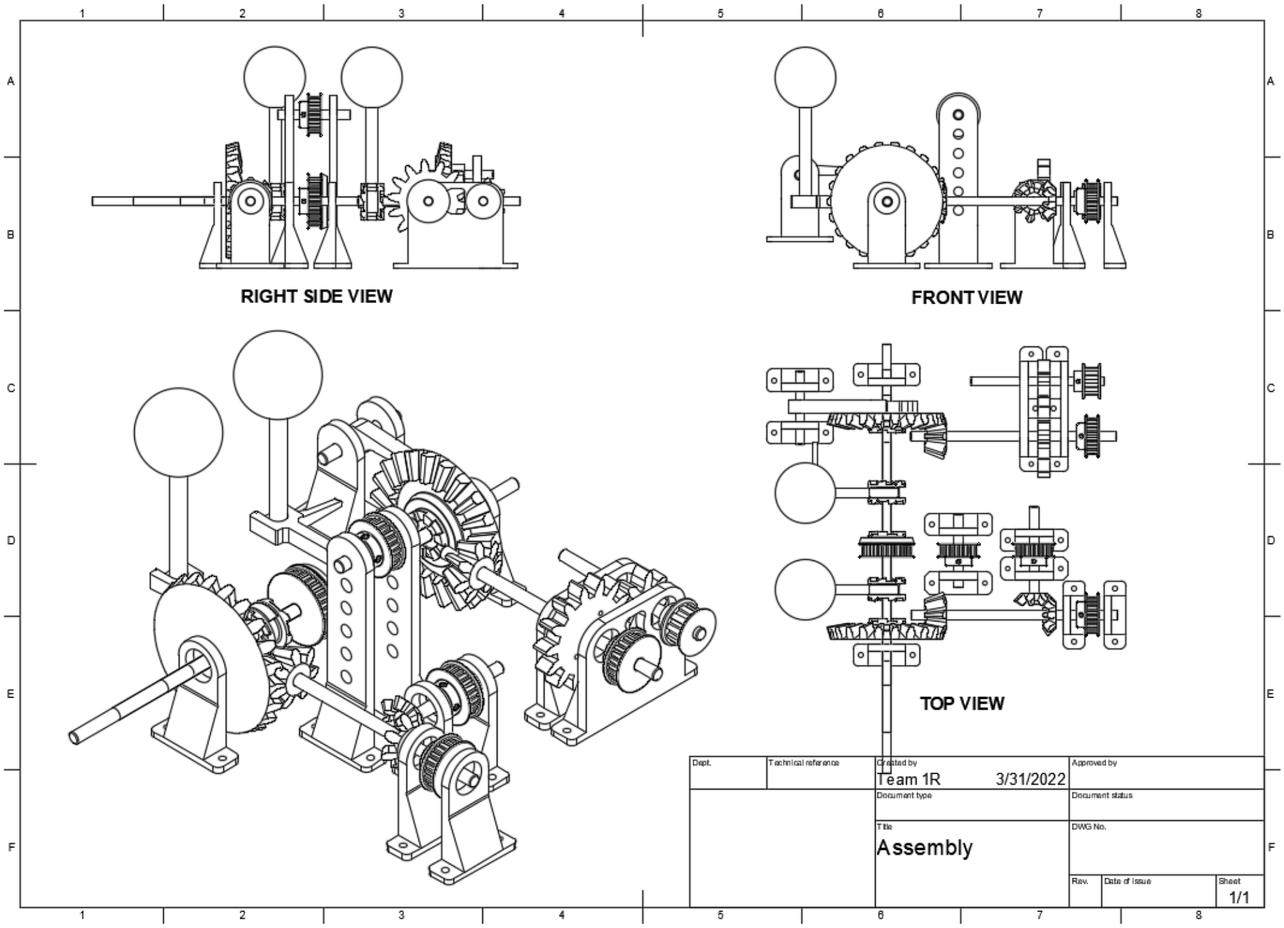

Figure 3. Fully Assembled Transmission CAD Drawing

Table 1. Gear Ratio and Lift Speed of Each Mode

Mode	Gear Ratio	Lift Speed (RPM)
High Speed	1:1.57	19.10
Low Speed	1:3.14	9.55
Reverse	1:3.14	9.55

Bill of Materials

The budget for this project was originally $100, with an additional $20 budget increase request granted part-way through the semester. The team worked hard to print custom parts at the various free printers around the UIUC campus. This allowed us to allocate most of our budget, towards store-bought materials. It is also worth noting that our team had many leftover materials from past projects that were recycled in the construction of our transmission. The actual weight may vary, as the resin density varies when cured. Table A1 in the appendix is an itemized list of the store-bought materials and custom parts. It is assumed that the density of resin used for the custom parts is 1.15 g/mL. To summarize, the team was able to come in $1.83 under the total $120 budget.

CAD Renderings of Custom Parts

Custom CAD was necessary for several aspects of this project. More specifically, spur gears, bevel gears, timing pulleys, dog clutches, shifting mechanism, ratcheting mechanism, and supports were all custom made for this transmission and can be seen in Figures 4 through 13. With our budget in mind, we chose to design the gears and clutches for 3D printing rather than machining. The three sets of 90-degree bevel gears were designed in CAD using Fusion's GF Gear Generator program, while the spur gears were designed by hand in Creo Parametric. A custom design for the shifting mechanism was necessary because of the unique dimensions of the shifting forks. Shaft supports were designed with the intention of adhering ball bearings to allow smooth rotation while the pawl supports and tensioner supports did not require this.

To determine the necessary gear ratios, we first calculated the input angular velocity using the diameter ratio of the shaft collar to the input timing pulley. Then, we found the output angular velocities needed to meet each of our time constraints and compared these angular velocities to the input velocity of our initial shaft. With these values, we calculated overall input-to-output gear ratios needed and gear diameters. This provided us with intermediate ratios between each meshing gear that we used to determine the number of teeth.

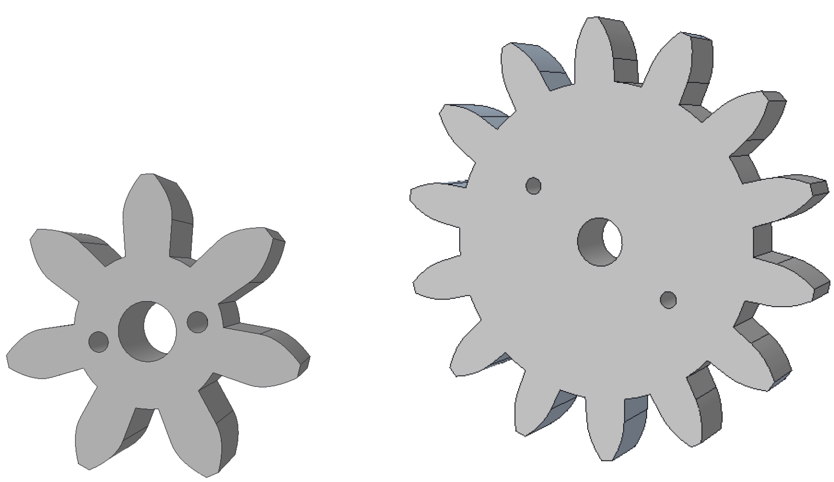

Figure 4. CAD Renderings of Spur A & B, Respectively

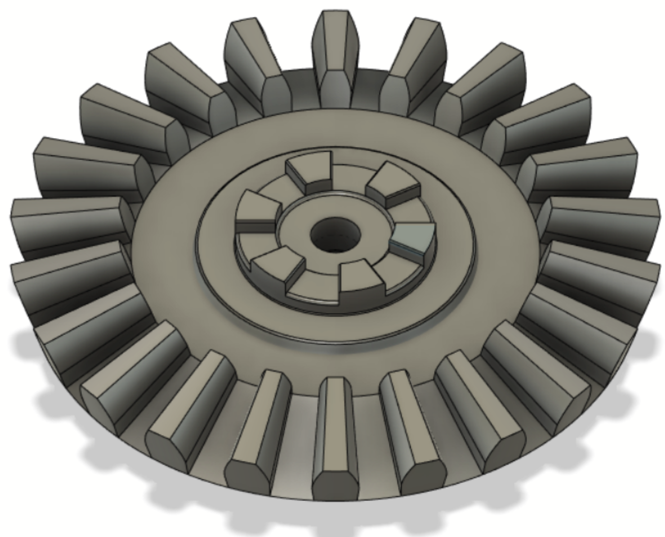

Figure 5. CAD Rendering of Bevel Gear A with Built-In Dog Clutch

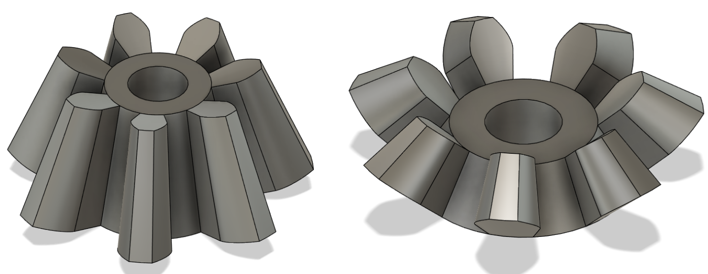

Figure 6. CAD Renderings of Bevel Gears B and C, Respectively

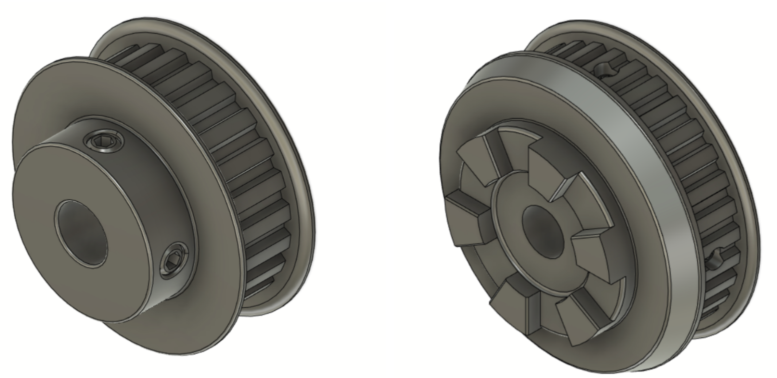

Figure 7. CAD Renderings of Timing Pulleys A and B, Respectively

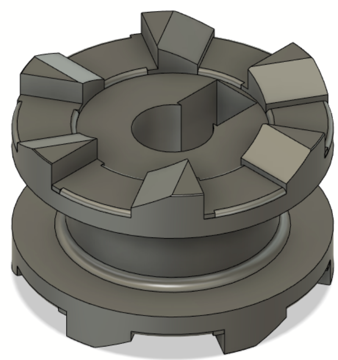

Figure 8. CAD Rendering of Dog Clutch

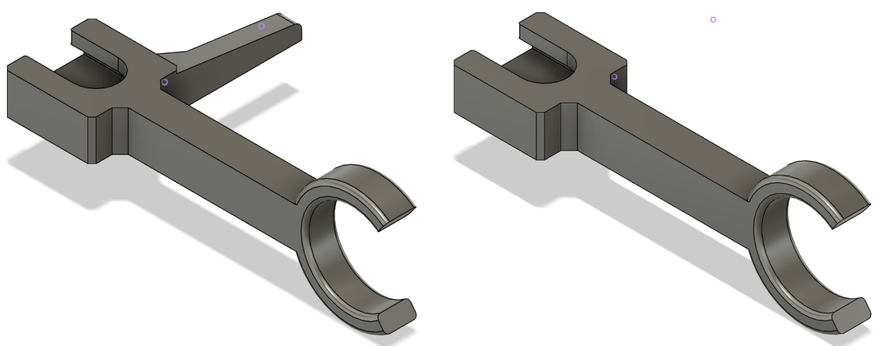

Figure 9. CAD Renderings of Shifting Fork A and B, Respectively

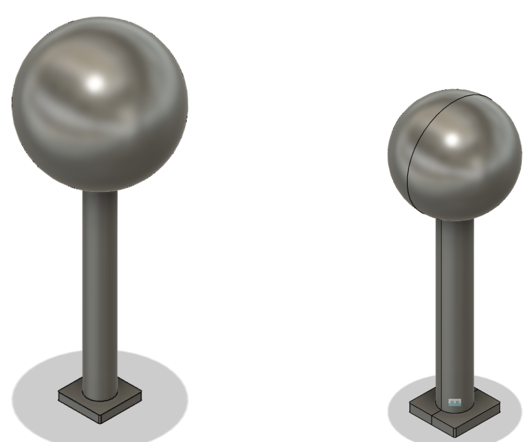

Figure 10. CAD Renderings of Shifting Knob A and B, Respectively

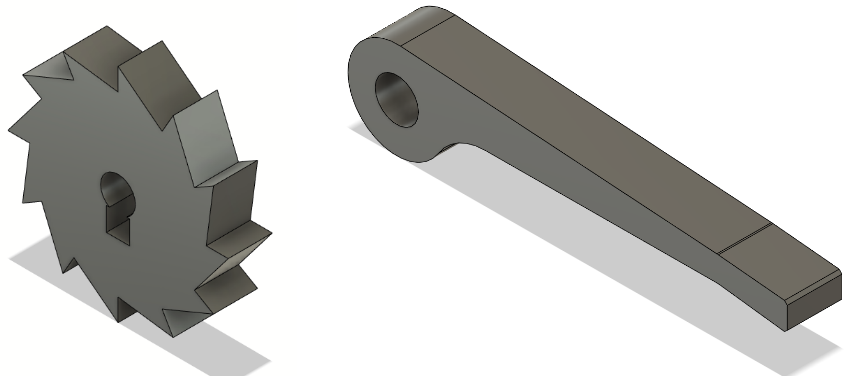

Figure 11. CAD Renderings of Ratchet Gear and Pawl

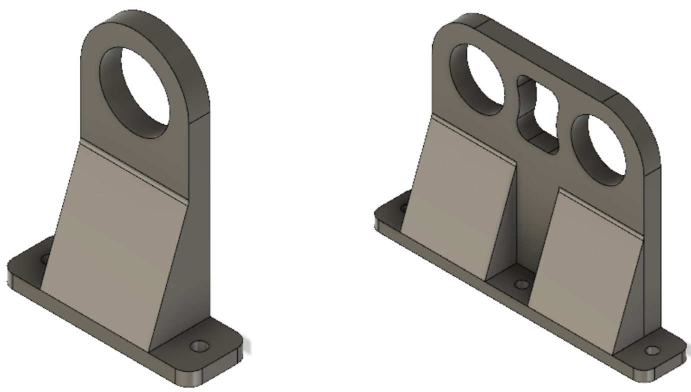

Figure 12. CAD Renderings of Single Supports and Double Supports

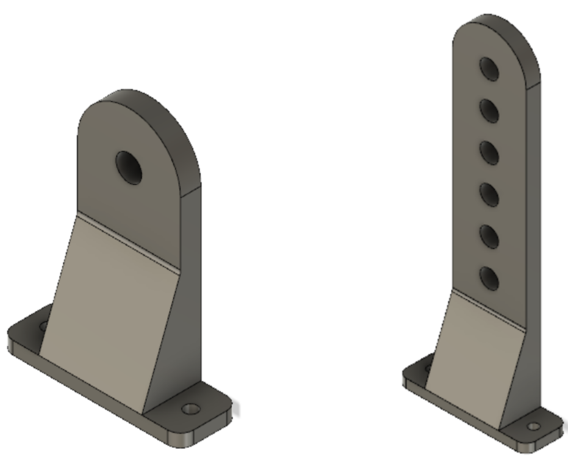

Figure 13. CAD Renderings of Pawl Support and Tensioner Support

Failure Analysis

Shaft Analysis

A shaft analysis was performed on the threaded circular shaft shown in Figure 14, containing gears 1, 4 and 14 while the transmission was operating at high speed. The corresponding force diagram can be seen in Figure 15.

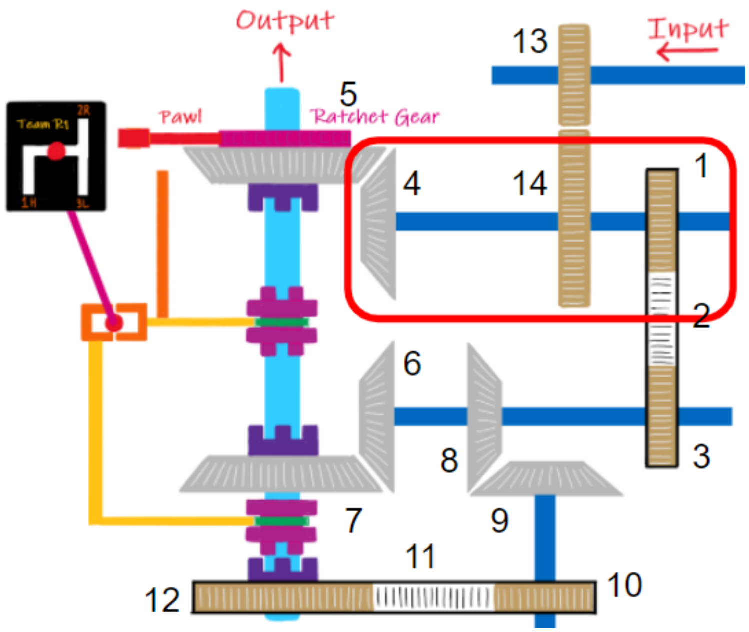

Figure 14. Numbered Schematic of Transmission

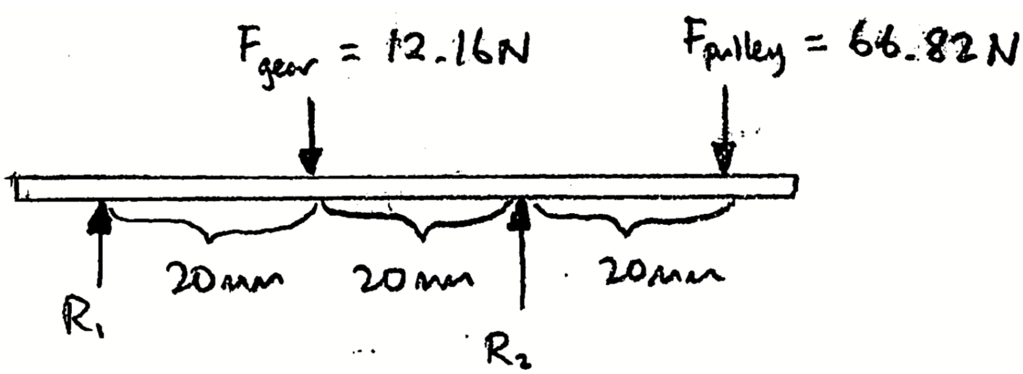

Figure 15. External Loads and Reaction Forces on the Shaft of Interest

Key assumptions for the shaft analysis include the following:

- Disengaged gears rotate freely and do not impose any loading on the shaft
- External loads and reaction forces from gears and ball bearings are point loads with no moment loads

- Threaded shaft assumed to have a circular cross section and any stress concentrations due to the threads are neglected
- Only the spur gear (14) and the pulley (1) are considered, since the bevel gear (4) does not drive any load when the transmission is in high speed

Using the values of R_1, R_2, $F_{gear,r}$, and F_{pulley}, calculated in Appendix A2.1, the shear and bending moment diagrams can be seen plotted in Figures 16 through 18.

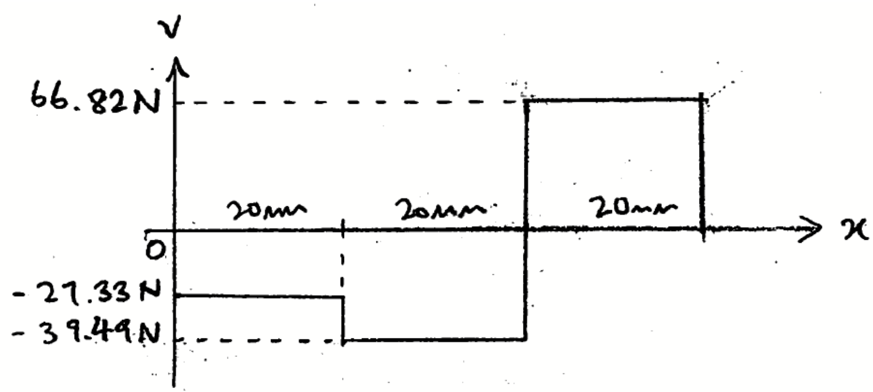

Figure 16. Internal Shear Diagram of the Shaft

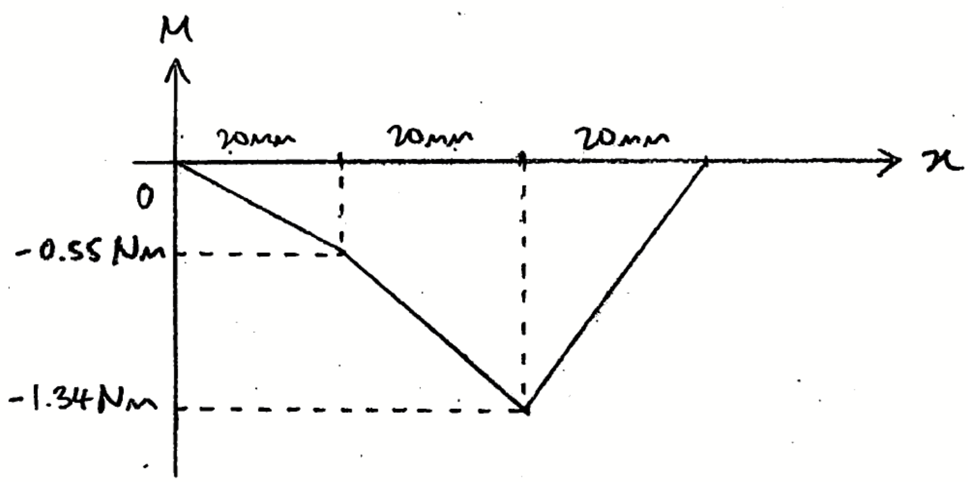

Figure 17. Bending Moment Diagram of the Shaft

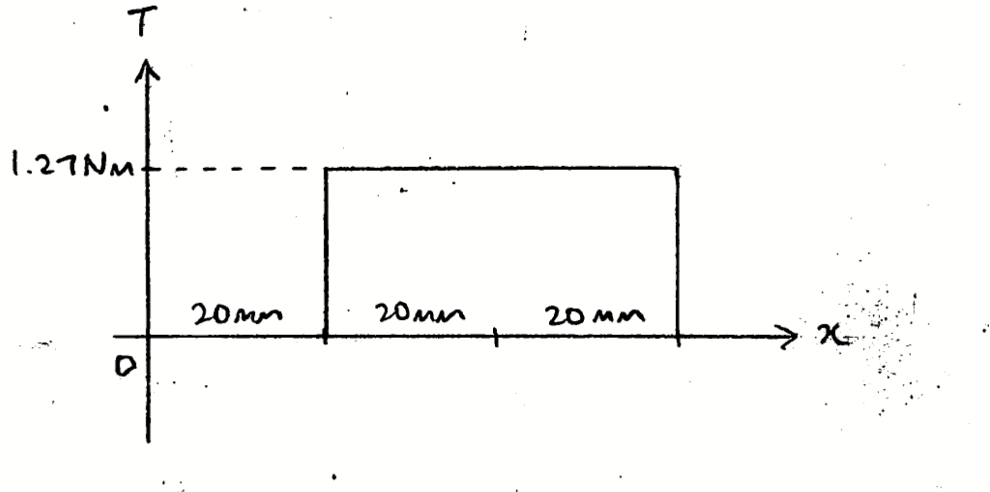

Figure 18. Torque Diagram of the Shaft

The maximum magnitude of the bending moment occurs at the point where reaction force R_2 is applied. This point also experiences the maximum magnitude of transverse shear. The maximum tensile stress due to the bending moment is calculated to be 15.79 MPa and occurs on the surface of the shaft at the length at which the bending moment is the greatest. The maximum shear stress due to torsion is calculated to be 7.50 MPa, occurs at the surface of the shaft, and is uniform lengthwise between the spur gear and pulley. The maximum shear stress due to transverse loading is calculated to be 1.25 MPa and occur at the rod's neutral axis. The calculated values for tensile stress and both sources of shear stress indicate that the maximum principal stress will exist on the surface of the shaft at the point where R_2 is applied. The principal stresses, σ_1 and σ_2, are 18.78 MPa and -2.99 MPa, respectively. Based on these resulst, the maximum Von-Mises stress and safety factor in the shaft are calculated as follows:

$$\sigma_{vm} = \sqrt{\sigma_1^{\,2} - \sigma_1\sigma_2 + \sigma_2^{\,2}} = 20.44\ MPa$$

$$SF = \frac{S_y}{\sigma_{vm}} = \frac{300}{20.44} = 14.7$$

While the exact material composition of the steel shaft is unknown, 300 MPa is a relatively conservative estimate for the yield strength S_y and results in a safety factor of 14.7. Therefore, the shaft is unlikely to fail even with the presence of stress concentrations. For reference, full calculations for intermediate values can be found in Appendix A2.1.

Machine Component Analysis

We conducted a machine component analysis on spur gear A. The material properties of the gear are assumed to be isotropic. Key assumptions for the machine component analysis include the following:

- ABS material properties (Mura, 2018), relatively similar to PLA; won't have significant effects on analysis conclusions
- No teeth sharing, 20-degree pressure angle: $J = 0.21$
- Low precision gears: $K_v = \frac{600+V}{600} = 1.04$
- Uniform power transfer: $K_o = 1$
- Low mounting accuracy: $K_m = 2.2$
- $b = 0.5$ in

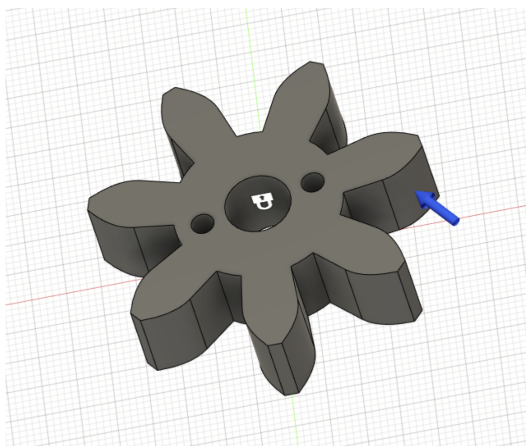

Figure 19. Free Body Diagram of Load on Spur Gear A

Figure 19 shows the free body diagram of spur gear A. Note that the external force shown is the net force F which consists of a tangential component F_t and a radial component F_r. These forces are linked via the following relationship:

$$F_r = F_t tan(\phi)$$

The spur gears have a pressure angle ϕ of 20 degrees. For the purpose of this analysis, the gear is assumed to be mounted rigidly to the shaft via the inner surface of the center hole. Due to the low dimensional tolerances of 3D printed parts, it is assumed that teeth sharing does not occur and, thus, the full load is applied to one tooth at a time. To find the tensile stress induced on spur gear A due to gear tooth bending, the pitch line velocity and load on the output shaft were calculated in Appendix A2.2. The power transferred by transmission is 0.00529 hp, the tangential force F_t on each tooth of the pinion is 7.41

lbs, and the gear pitch is 4.67 in^{-1}. Based on these values, the tensile stress induced on spur A is calculated as follows:

$$\sigma = \frac{F_t P}{bJ} K_v K_o K_m = \frac{7.41(4.67)}{0.5(0.21)} (1.04)(2.2) = 0.754\ ksi$$

We also conducted a gear-tooth fatigue analysis on spur A to determine its endurance limit. Key assumptions for the fatigue analysis include the following:

- $C_L = 1.0$
- $P < 5$: $C_G = 0.85$
- Conservative estimate due to rough surface finish: $C_S = 0.3$
- 99% reliability: $k_r = 0.814$
- $k_t = 1.0$
- Non-idler gear: $k_{ms} = 1.4$
- Assume ABS material properties for endurance limit (Mura, 2018): $S_n' = 37.3$ MPa = 5.41 ksi

We then used the following equations to calculate the endurance limit and safety factor for spur gear A:

$$S_n = S'_n C_L C_G C_S k_r k_t k_{ms} = 1.57\ ksi$$

$$\text{Safety factor: } SF = \frac{S_n}{\sigma} = \frac{1.57}{0.754} = 2.08$$

Since $S_n > \sigma$, we know that spur gear A is unlikely to fail due to fatigue. For reference, full calculations for intermediate values can be found in Appendix A2.2.

FEA Stress Analysis

We conducted an FEA analysis on Dog Clutch A in order to determine its strength and the location of highest stress concentrations. The free body diagram, as seen in Figure 24, outlines the constraints and external loads applied to the component. To constrain the model, we created a fixed constraint on the flat face and rounded surface of the d-shaft hole. We also distributed a 105 N load across the six vertical dog clutch teeth surfaces to simulate the torque and stresses acting on the part when driving the output shaft. Adaptive Mesh Refinement (AMR) was selected during mesh generation to improve simulation accuracy, especially around complex geometries. Figure 23 shows the resulting mesh generated by Fusion 360's AMR function. The total external loading on the dog clutch surfaces was determined as follows:

$$T_{output} = \frac{D_{pulley} m_{load} g}{2} = \frac{(4)(25.4)(4)(9.81)}{2} \approx 2000\ Nmm$$

$$Load_{total} = \frac{2T_{output}}{D_{clutch}} = \frac{2(20000)}{1.2(25.4)} \approx 130\ N$$

The dog clutch diameter D_{clutch} is the average diameter between the inner and outer diameters of the clutch teeth. The total external loading, 130 N, was assumed to be evenly distributed across all six teeth. Since Fusion 360 did not have PLA plastic in its built-in material properties database, ABS plastic was selected due to its relatively similar material properties.

As seen in Figure 20, we found that the maximum Von Mises stress was 5.963 MPa, which occurred at the base of the dog clutch walls. The maximum displacement was 0.01663 mm, which can be seen in the 2X adjusted plot in Figure 21. The minimum safety factor against yielding was 2.709, as seen in Figure 22. The part is at risk of failing where the teeth of the dog clutch mesh with Bevel Gear A, at both low and reverse speeds. To improve the performance of the dog clutch, we can modify the part geometry by increasing thickness and width of the teeth, or adding a fillet to the sharp corners of the teeth to prevent stress concentrations from forming. Overall, the FEA simulation showed that Dog Clutch A can withstand the loads placed on it by the other transmission components.

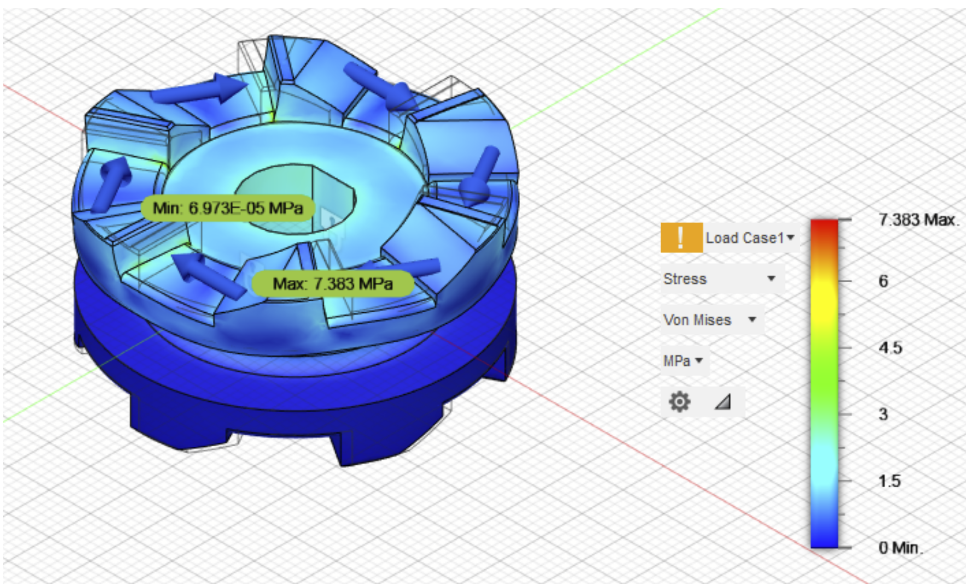

Figure 20. Maximum and Minimum Von Mises Stress on Dog Clutch A

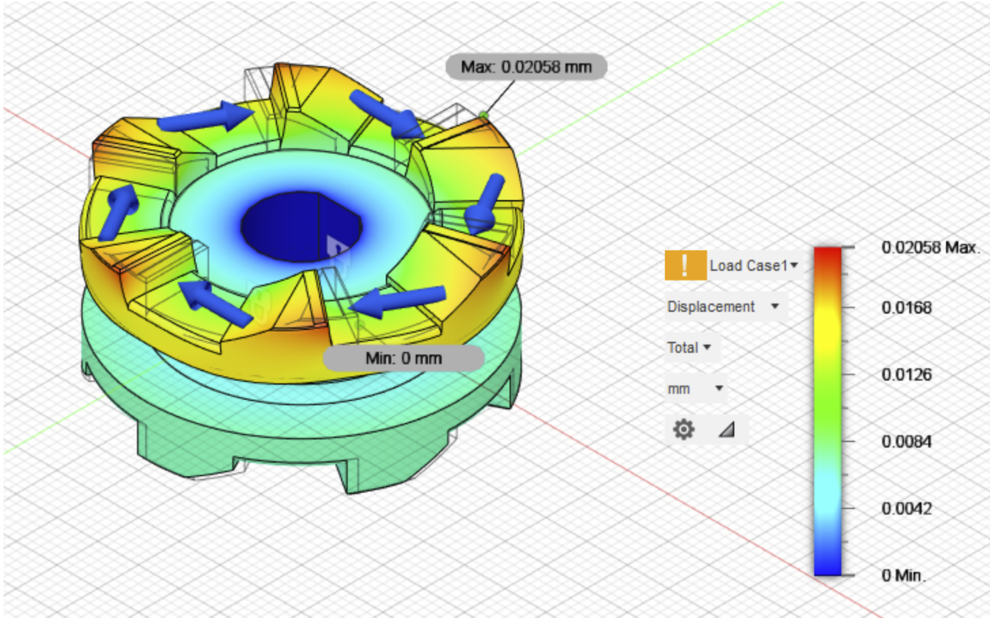

Figure 21. 2X Adjusted View of Minimum and Maximum Displacement

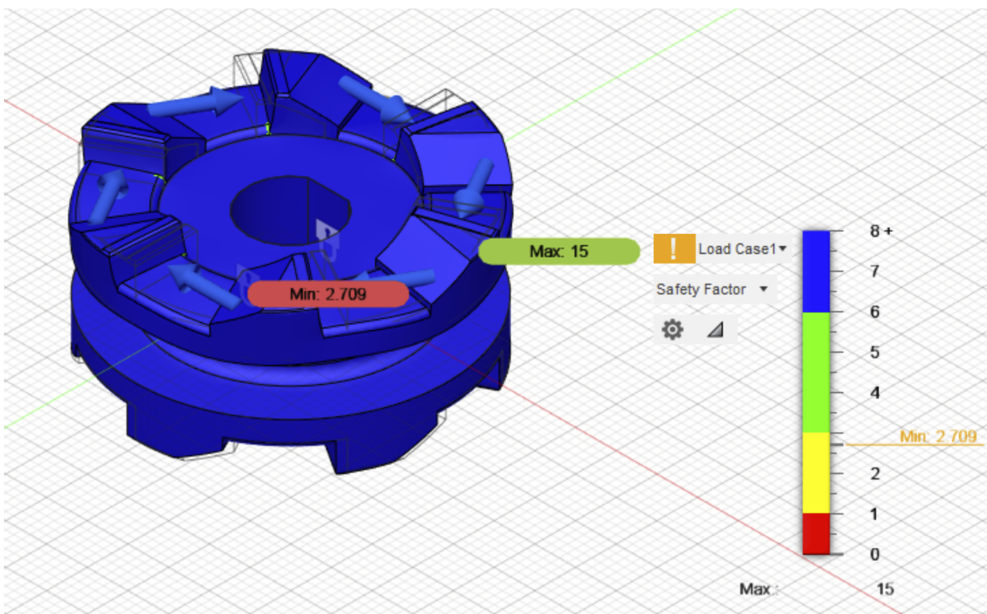

Figure 22. Maximum and Minimum Safety Factors

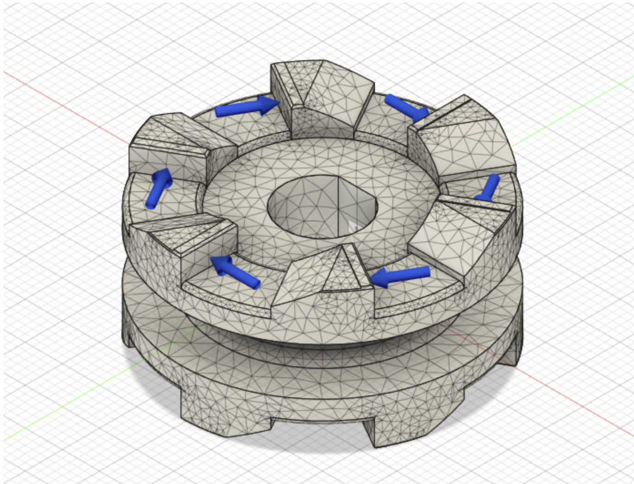

Figure 23. Final AMR Mesh Generated in Fusion 360

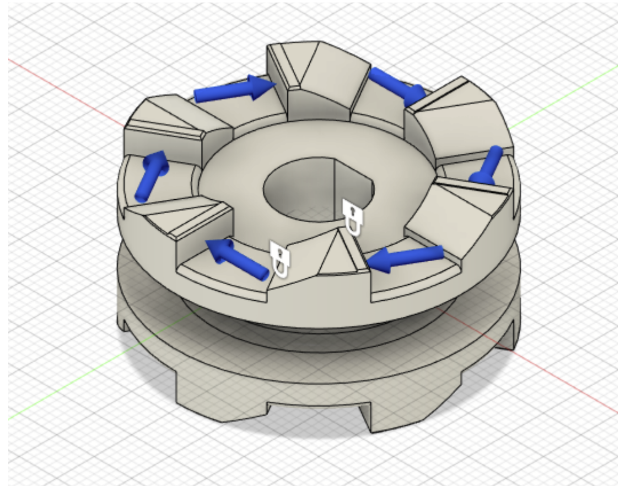

Figure 24. Free Body Diagram of Loads & Constraints

Reflection

Our team was able to successfully raise and lower a 4 kg weight using all three transmission speeds. Some challenges our team encountered that we did not anticipate were staying under budget with so many 3D printed part iterations, getting our design to shift smoothly, making sure our dog clutches were rigidly connected to our d-shaft, and working with an oversized belt due to limited sizes available through McMaster Carr. To overcome the challenge of staying under budget, we did our best to utilize all the free 3D printing facilities on campus such as ECE Open Lab and Siebel Center for Design. To help our design shift smoothly, we added lubricant to the shifting arms and dog clutch connections, and made the arms and clutch spacing thicker to prevent the parts from cracking under a large load. To prevent the dog clutches from slipping on the shaft when faced with high torque, we used JB Weld to permanently secure a rectangular key to the flat face of the d-shaft and designed a corresponding slot in the clutch. To allow our design to fit within proper size constraints and create more tension in our oversized belt, we decided to attach a third timing pulley at a greater height within the high speed gear-belt mechanism. We also custom designed tall supports to hold the third timing pulley in place, with multiple holes to allow us to test which height provided the correct amount of tension in the belt.

During our final benchmark, we found that our ratchet gear and pawl mechanism sometimes slipped when switching speeds, causing the bucket to slightly lower in idle. This caused our shifting motion to be a bit more erratic than predicted. The meshing between both Bevel C gears also slipped due to high torque; this caused the high speed mechanism to be slower than expected and move in a staggered manner. Other than these caveats, our transmission design worked well and was able to lift and lower the 4 kg weight in a controlled manner at all three speeds. Our low speed mechanism worked particularly well, as it was able to disengage the ratchet gear and pawl using the shifting arm while lifting the weight within the allotted time constraint. If we had the opportunity to further improve our design, we would 3D print a rail system for the shifting arms to move smoothly, add a rectangular key to the ratchet gear in a similar manner to the dog clutches, and design the Bevel C gears with slightly larger teeth to prevent any slipping. Overall, our team had a great experience working on this project together, and we were able to build a successful transmission design.

Appendix

A1. Itemized Bill of Materials Table

Table A1. Bill of Materials Divided by Store Bought and Custom Parts

Store Bought				
Item	**Qnty**	**Dimensions / Notes**	**Cost ($)/Unit**	**Total Cost ($)**
Belt A	1	18" circumference, 3/8" width	8.42	8.42
Belt B	1	20" circumference, 3/8" width	8.67	8.67
Threaded Intermediate Shaft	1	3/8" diameter, 1 m long	9.40	9.40
Ball Bearing	10	3/8" inner diameter	6.46	64.60
J-B Weld	1	Epoxy	8.29	8.29
Loctite Red	1	Unused	8.29	8.29
Innovation Studio Punch Card	1	Shaft collars and allen keys		10.50
			Total Spent	**$118.17**

Custom Parts					
Item	**Qnty**	**Dimensions / Notes**	**Material**	**Weight (g)/Unit**	**Total Weight (g)**
Spur A	1	7 teeth, 1.5" pitch diameter	PLA	9.0	9.0
Spur B	1	14 teeth, 3" pitch diameter	PLA	29.1	29.1

Timing Pulley A	2	24 teeth, 1.5" pitch diameter	PLA	12.7	25.4
Timing Pulley A	2	24 teeth, 1.5" pitch diameter	Resin	23.5	47.0
Timing Pully B	1	32 teeth, 2" pitch diameter	Resin	33.6	33.6
Dog Clutch	2	3/8" inner diameter	Resin	14.0	28.1
Bevel Gear A with Built-In Dog Clutch	2	22 teeth, 4.71" pitch diameter	PLA	66.0	132.0
Bevel Gear B	1	7 teeth, 1.5" pitch diameter	PLA	11.0	11.0
Bevel Gear B	1	7 teeth, 1.5" pitch diameter	Resin	17.4	17.4
Bevel Gear C	2	7 teeth, 1.5" pitch diameter	Resin	7.0	14.1
Shifting Fork A	1	Built in pawl engagement	Resin	21.8	21.8
Shifting Fork B	1		Resin	20.7	20.7
Shifting Knob A	2	Half of full knob	Resin	86.5	173.1
Shifting Knob B	2	Smaller version of A	Resin	33.8	67.6
Ratchet	1		PLA	16.4	16.4
Pawl	1		PLA	8.4	8.4
Single Support	6	With hole for ball bearings	PLA	22.8	136.8
Double Support	2	With hole for ball bearings	PLA	49.7	99.4
Pawl Support	2		PLA	24.2	48.4
Tensioner Support	2		PLA	39.8	79.6
				Total Weight	**1,018.76 g**

A2. Failure Analysis Calculations

A2.1. Shaft Analysis Calculations

$$T_{output} = \frac{D_{pulley} m_{load} g}{2} = \frac{(4)(25.4)(4)(9.81)}{2} \approx 2\ Nm$$

$$Gear\ ratio\ K_g\ from\ output\ to\ shaft\ = \frac{7}{11}$$

$$T_{shaft} = T_{output} K_g = 2(\frac{7}{11}) = 1.273\ Nm$$

$$D_{gear} = 3\ in = 76.2\ mm$$

$$D_{pulley} = 1.5\ in = 38.1\ mm$$

$$F_{gear,t} = \frac{T_{shaft}}{R_{gear}} = \frac{1.273}{(\frac{0.0762}{2})} = 33.41\ N$$

$$F_{gear,r} = F_{gear,t} tan(\phi) = 33.41 tan(20) = 12.16\ N$$

$$F_{pulley} = \frac{T_{shaft}}{R_{pulley}} = \frac{1.273}{(\frac{0.0381}{2})} = 66.82\ N$$

Solve for the bearing reaction forces R_1 and R_2 using the sum of moments and sum of forces:

$$\Sigma M = 0.04R_2 - 0.02F_{gear,r} - 0.06F_{pulley} = 0$$

$$R_2 = 106.31\ N$$

$$\Sigma F = R_1 + R_2 - F_{gear,r} - F_{pulley} = 0$$

$$R_1 = -27.33\ N$$

Solve for the maximum tensile stress due to the bending moment:

$$\sigma = \frac{M(\frac{d}{2})}{\frac{\pi}{64}d^4} = \frac{32M}{\pi d^3} = \frac{32(1.34)}{\pi(\frac{9.525}{1000})^3} = 15.79\ MPa$$

Solve for the maximum shear stress due to torsion:

$$\tau_{torsion} = \frac{T(\frac{d}{2})}{\frac{\pi}{32}d^4} = \frac{16T}{\pi d^3} = \frac{16(1.273)}{\pi(\frac{9.525}{1000})^3} = 7.50\ MPa$$

Solve for the maximum shear stress due to transverse loading:

$$\tau_{transverse\ shear} = \frac{4}{3}\frac{V}{\frac{\pi}{4}d^2} = \frac{16V}{3\pi d^2} = \frac{16(66.82)}{3\pi(\frac{9.525}{1000})^2} = 1.25\ MPa$$

Solve for the principal stresses $\sigma_1$and σ_2:

$$\sigma_1 = \frac{\sigma_x+\sigma_y}{2} + \sqrt{\tau_{xy}^{\ 2} + (\frac{\sigma_x-\sigma_y}{2})^2} = \frac{15.79}{2} + \sqrt{7.50^2 + (\frac{15.79}{2})^2} = 18.78\ MPa$$

$$\sigma_2 = \frac{\sigma_x+\sigma_y}{2} - \sqrt{\tau_{xy}^{\ 2} + (\frac{\sigma_x-\sigma_y}{2})^2} = \frac{15.79}{2} - \sqrt{7.50^2 + (\frac{15.79}{2})^2} =- 2.99\ MPa$$

Solve for Von-Mises stress and safety factor in the shaft:

$$\sigma_{vm} = \sqrt{\sigma_1^{\ 2} - \sigma_1\sigma_2 + \sigma_2^{\ 2}} = 20.44\ MPa$$

$$SF = \frac{S_y}{\sigma_{vm}} = \frac{300}{20.44} = 14.7$$

A2.2. Machine Component Analysis Calculations

Solve for pitch line velocity of 7 tooth pinion:

$$V = \frac{\pi dn}{12} = \frac{\pi(1.5)(60)}{12} = 23.56\ ft/min$$

Solve foroad on output shaft:

$$F_{load} = 4\ kg = 8.82\ lb$$

Solve for power transferred by transmission:

$$\dot{W} = \frac{F_{load}V}{33000} = \frac{8.82(0.33)(60)}{33000} = 0.00529\ hp$$

Solve for force on each tooth of the pinion:

$$F_t = \frac{33000\dot{W}}{V} = 7.41\ lbs$$

Solve for diametral pitch:

$$P = \frac{N}{d} = \frac{7}{1.5} = 4.67$$

Solve for tensile stress due to gear tooth bending:

$$\sigma = \frac{F_t P}{bJ} K_v K_o K_m = \frac{7.41(4.67)}{0.5(0.21)} (1.04)(2.2) = 0.754\ ksi$$

Fatigue Analysis:

Solve for endurance limit and safety factor:

$$S_n = S'_n C_L C_G C_S k_r k_t k_{ms} = 1.57\ ksi$$

$$SF = \frac{S_n}{\sigma} = \frac{1.57}{0.754} = 2.08$$

References

1. Mura, A., Ricci, A., & Canavese, G. (2018). Investigation of Fatigue Behavior of ABS and PC-ABS Polymers at Different Temperatures. *Materials (Basel, Switzerland)*, *11*(10), 1818. https://doi.org/10.3390/ma11101818