ME 320 Heat Transfer Experiment: Convection of a Finned Surface with Varied Fin Length and Base Dimensions

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Introduction

Some surfaces contain fins that increase surface area and therefore support the cooling of a device by increasing the convection heat transfer rate from the surface. These finned surfaces, particularly heat sinks, are commonly used to cool devices that generate heat but are required to operate at a low temperature to avoid damage due to overheating. Many electronic devices use heat sinks to enhance heat transfer from the device and maintain the operating temperature below a specific level. Finned heat sinks accomplish this as highly conductive materials that significantly add to the surface area of the system. The greater the surface area, the higher the rate of heat transfer due to convection. This experiment quantifies how much different finned surface properties impact this heat transfer.

Research Question: How much does fin length and base surface area of a finned surface affect the convection heat transfer rate from this surface?

Experimental Methods

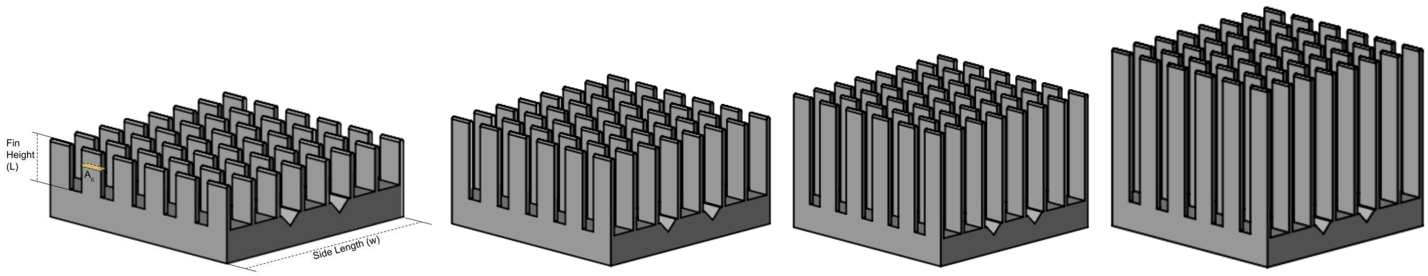
Evaluating the convection heat transfer rate of a finned surface in this experiment requires several heat sinks of a uniform material with different fin lengths and base surface areas. One hot plate is used to heat each of the specimens with a constant temperature and contact to the bottom surface, while a thermocouple is used to measure the resulting steady-state surface temperature of the side exposed to air.

Materials Needed:

- 4 finned heat sinks of the same base surface area with different fin heights (anodized aluminum)
- 3 finned heat sinks of the same fin height with different base surface areas (anodized aluminum)
- Hot plate to apply a controlled constant surface temperature
- Thermocouple
- Temperature monitor
- Spreadsheet to record steady-state temperature values of each surface's fin tip

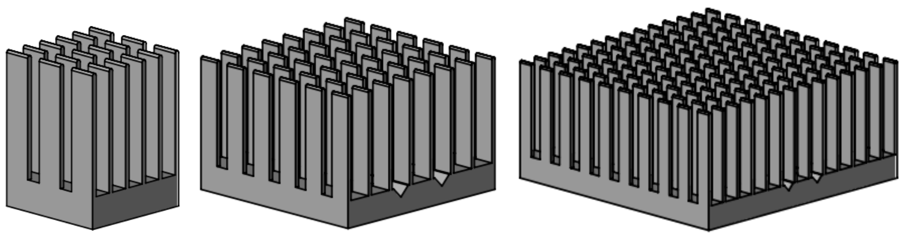
Experimental Setup:

Figures 1 and 2 show each of the anodized aluminum heat sink specimens used in this experiment.



1 **2** **3** **4**

Figure 1: Specimens 1, 2, 3, and 4 with Varied Fin Heights



5 **3** **6**

Figure 2: Specimens 5, 3, and 6 with Varied Base Surface Area

The steady-state temperature of each specimen's fin tip and air-exposed surface are measured using the experimental setups shown in Figures 3 and 4 with each specimen placed on the same hot plate to provide a constant temperature at the contact point.

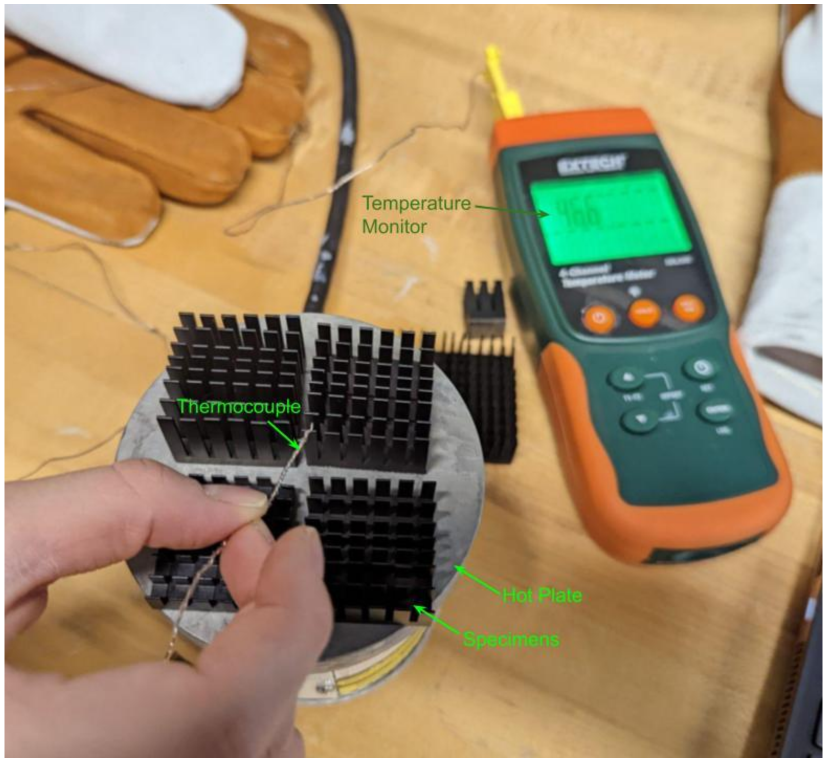


Figure 3: Experimental Setup of Specimens with Different Fin Lengths

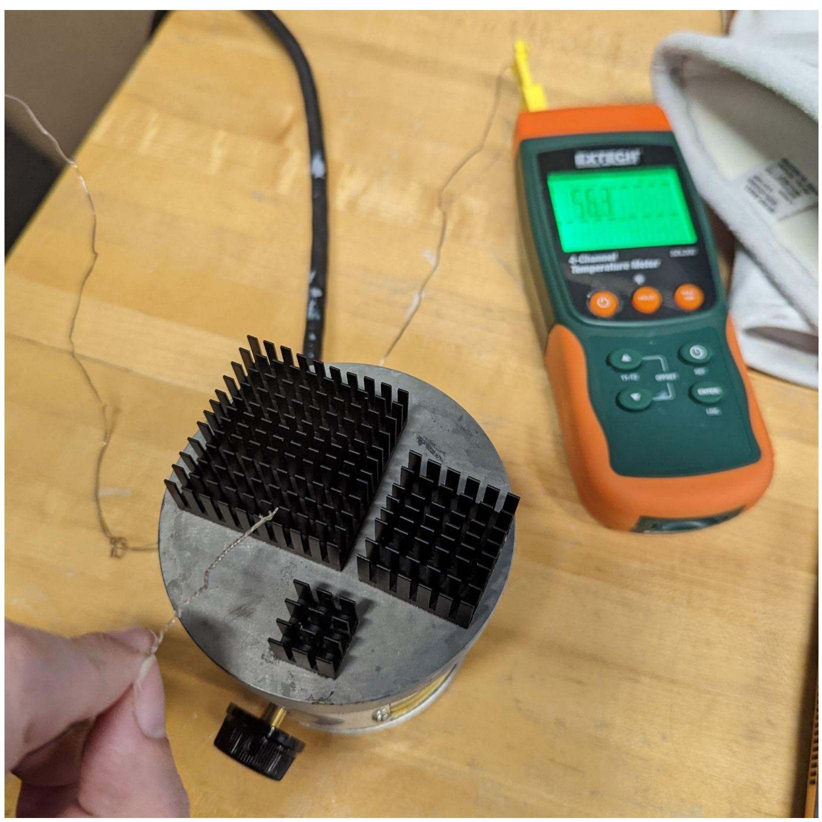


Figure 4: Experimental Setup of Specimens with Different Base Surface Areas

Experimental Procedure:

1. Record the room temperature
2. Turn on the hot plate to a constant power level and maintain throughout the experiment
3. Use a thermocouple to monitor the surface temperature of the hot plate and record this temperature when it reaches a steady state
4. Place all four heat sinks of different fin heights on the hot plate
5. Collect steady-state temperature data
    a. For each specimen, place the thermocouple on the base and record the steady temperature
    b. For each specimen, place the thermocouple on one of the fin tips and record the steady temperature
6. Calculate the theoretical convection heat transfer rate of each specimen using its defined thermal resistance and the measured difference between surface temperature and room temperature
7. Calculate the experimental convection heat transfer rate from the experimental results as described by the Calculation Methods section
8. Repeat this procedure for evaluation of the three heat sinks with different base surface areas

Assumptions:

1. Steady-state conditions
2. Radiation is negligible and resistance due to conduction is negligible
3. Surface temperature at the bottom of each specimen due to contact with the hot plate is equal and constant
4. No contact thermal resistance between the hot plate and heat sinks
5. Uniform convection heat transfer coefficient of anodized aluminum

Calculation Methods:

Thermal resistance of a finned surface is represented by Equation 1 while this equation solved for the convection heat transfer rate is represented by Equation 2.

Equation 1:

$$R_{t,0} = \theta_b \,/\, q_t = (T_b - T_\infty) \,/\, q_t$$

Equation 2:

$$q_t = (T_b - T_\infty) \,/\, R_{t,0}$$

The convection heat transfer rate of each finned surface can also be calculated using Equation 3.

Equation 3:

$$q_t = \eta_0 h A_t \theta_b = \eta_0 h A_t (T_b - T_\infty)$$

Given this relation, we can substitute Equation 3 into Equation 1 to create a new equation for thermal resistance that defines this value in terms of the convection coefficient, total surface area, and overall surface efficiency.

Equation 4:

$$R_{t,0} = \theta_b / q_t = \theta_b / \eta_0 h A_t \theta_b = 1 / \eta_0 h A_t$$

The overall surface efficiency is calculated using Equation 5.

Equation 5:

$$\eta_0 = q_t / q_{max} = 1 - (N A_f / A_t)(1 - \eta_f)$$

Calculating the overall surface efficiency requires the fin efficiency, which can be found using Equations 6 and 7 below.

Equation 6:

$$\eta_f = tanh(mL_c) / mL_c$$

Equation 7:

$$m = (hP / kA_c)^{1/2}$$

Total surface area A_t is also needed to calculate the overall surface efficiency and must be factored into the thermal resistance formula. This can be found by applying known specimen dimensions to the equations below. Fin thickness is represented by t, fin width is represented by w, and its length is represented by L. The base of each specimen is square, so the surface width and height (H_{wall}) are equal. N represents the number of fins on the surface.

Equation 8:

$$L_c = L + (t/2)$$

Equation 9:

$$A_f = PL_c = 2(t + w)L_c$$

Equation 10:

$$A_c = wt$$

Equation 11:

$$A_b = H_{wall}^2 - NA_c$$

Equation 12:

$$A_t = A_b + NA_f$$

While thermal resistance as well as these dimensional parameters needed to calculate overall surface efficiency and total surface area are known, the convection coefficient of each specimen is unknown. This is the only unknown parameter in Equation 4, so we can use Equation 4 to solve for the convection coefficient and then use this coefficient to evaluate the convection heat transfer rate and heat flux of each specimen. Equation 13 represents Equation 4 with the overall surface efficiency equations factored in.

Equation 13:

$$R_{t,0} = 1 / (1 - (NA_f / A_t)(1 - (tanh((hP / kA_c)^{1/2}L_c) / (hP / kA_c)^{1/2}L_c)))hA_t$$

Solving for the convection coefficient h is not a direct calculation, but since the other variables are known, MATLAB can be used to solve Equation 13 for the convection coefficient that yields the known thermal resistance. Once this convection coefficient is found, it is factored into Equation 3 along with the experimental temperature data to find the heat transfer rate and heat flux due to convection at different finned surfaces.

Results and Discussion

Several fin dimensions and calculations are needed for each heat sink specimen to perform the experimental convection heat transfer rate calculations. These dimensions, calculated values, and defined thermal properties are shown in Table 1 below.

Table 1: Finned Heat Sink Specimen Dimensions and Known Properties

Specimen #	Measured Dimensions						Dimension Calculations						Thermal Resistance (C/W)	Conduction Coefficient (W/m K)
	Fin Height (mm)	Base Side Length (mm)	Base Surface Area (mm^2)	Fin Thickness (mm)	Fin Width (mm)	# of Fins	P (mm)	L_c (mm)	A_c (mm^2)	A_f (mm^2)	A_b (mm^2)	A_t (mm^2)		
1	9.5	30	900	0.61	3.30	48	7.82	9.81	2.01	76.68	803.38	4483.78	14.0	1.12
2	14.5	30	900	0.56	3.36	48	7.84	14.78	1.88	115.88	809.68	6371.69	7.6	1.12
3	19.5	30	900	0.76	3.31	48	8.14	19.88	2.52	161.82	779.25	8546.76	5.3	1.12
4	24.5	30	900	0.79	3.49	48	8.56	24.90	2.76	213.10	767.66	10996.52	4.1	1.12
5	19.5	15	225	0.77	3.57	12	8.68	19.89	2.75	172.60	192.01	2263.23	15.1	1.12
6	19.5	45	2025	0.73	2.75	120	6.96	19.87	2.01	138.26	1784.10	18375.35	3.0	1.12

<u>Varied Fin Height Results</u>:

The MATLAB code in Appendix A applies known values to Equation 13 to find the convection coefficients for the first four specimens of different fin heights. These convection coefficients are shown in the results—Tables 2 and 3—and applied to calculation of the convection heat transfer rate and heat flux of each specimen.

Table 2: Convection Heat Transfer Rate Results for Different Fin Lengths

	Specimen #	**1**	**2**	**3**	**4**
Experimental Data	Height (mm)	9.5	14.5	19.5	24.5
	Hot Plate Temp (C)	91.1	91.1	91.1	91.1
	Surface Temp (C)	85.4	81.9	78.7	76.5
	Top Fin Temp (C)	76.3	73.2	69.6	62.8
	Room Temp (C)	21.0	21.0	21.0	21.0
Convection Coefficient (W/m^2 K)		88.21	161.10	239.21	313.21
	m	553.54	774.49	830.60	931.30
	n_f	0.18	0.09	0.06	0.04
	n_0	0.33	0.20	0.15	0.11
Experimental Convection Heat Transfer Rate (W)		8.41	12.71	17.25	21.01
Experimental Convection Heat Flux (W/m^2)		1876.17	1994.57	2018.33	1910.88

During this experiment, we observed that the surface temperature of the finned heat sink decreases as the fin length increases and therefore the total surface area A_t increases. This means a decreasing difference between the surface temperature and the temperature of the surroundings that factors into the experimental heat transfer rate calculation. These results are consistent with the expectation that heat loss will be less concentrated—or efficient—the larger the total area of the surface. The experimental heat transfer rate increases with increasing fin length, because although a greater total surface area decreases the fin and overall efficiency, the increase in surface area has more of an impact on the calculated heat transfer rate than decreasing efficiency.

Heat flux, or the rate of heat transfer per unit area, however, experiences minimally significant change. This suggests that the decrease in overall efficiency is balanced out by the increasing calculated convection coefficient.

Varied Base Surface Area Results:

Similar to the results for different fin heights in Table 2, Table 3 shows the calculated convection coefficients, convection heat transfer rate, and heat flux of the specimens with different base side lengths.

Table 3: Convection Heat Transfer Rate Results for Different Base Side Lengths

	Specimen #	5	3	6
Experimental Data	Side Length (mm)	15.0	30.0	45.0
	Hot Plate Temp (C)	115.8	115.8	115.8
	Surface Temp (C)	107.5	91.2	86.5
	Top Fin Temp (C)	94.7	83.2	78.7
	Room Temp (C)	21.0	21.0	21.0
Convection Coefficient (W/m^2 K)		341.23	239.21	184.56
	m	980.64	830.60	755.38
	n_f	0.05	0.06	0.07
	n_0	0.13	0.15	0.16
Experimental Convection Heat Transfer Rate (W)		8.80	20.99	34.93
Experimental Convection Heat Flux (W/m^2)		3889.29	2455.58	1900.95

For specimens of varied base surface area, we observed that increasing base surface area decreases the surface temperature and the difference between the surface temperature and surroundings similar to an increase in fin length. This is also expected behavior because greater base surface area increases the total surface area A_t. The fin and overall efficiencies increase slightly between these specimens, however, unlike the behavior they experience due to increasing fin length. Based on this, it seems that a greater ratio of fin length to base area decreases efficiency. In the fin length analysis, we can see that this ratio increases because the fin length increases with a constant base surface area and it correlates to decreasing efficiency. The ratio decreases with the increasing base surface area and constant fin length where we see an increasing overall and fin efficiency. The calculated heat transfer rate increases with greater base surface area, which is expected due to the greater change in total surface area compared to its relatively small change in the fin height analysis. The other factor that seems to contribute to a more significant increase in heat transfer rate is the increasing overall efficiency. Unlike with the fin length change, the heat flux of specimens with different base surface areas experiences significant change. The larger the surface area, the less concentrated the heat transfer is from the surface.

Convection Coefficient and Heat Transfer Rate Results Comparison:

We can see from Equation 2 that the manufacturer-defined thermal resistance and the experimental temperature data can determine a theoretical convection heat transfer rate to compare to the calculated experimental heat transfer rate. These theoretical values are shown with the experimental results in Table 4.

Table 4: Results Summary and Error Calculation

	Varied Fin Heights				Varied Base Side Length / Surface Area		
Specimen #	**1**	**2**	**3**	**4**	**5**	**3**	**6**
Height / Base Side Length (mm)	9.5	14.5	19.5	24.5	15.0	30.0	45.0
Theta_b (C)	64.40	60.90	57.70	55.50	86.50	70.20	65.50
Convection Coefficient	88.21	161.10	239.21	313.21	341.23	239.21	184.56
Experimental Heat Flux	1876.17	1994.57	2018.33	1910.88	3889.29	2455.58	1900.95
Experimental Heat Transfer Rate (W)	8.41	12.71	17.25	21.01	8.80	20.99	34.93
Theoretical Heat Transfer Rate (W)	4.60	8.01	10.89	13.54	5.73	13.25	21.83
Heat Transfer Rate Error (%)	82.83	58.68	58.40	55.17	53.58	58.42	60.01

Uncertainty Analysis:

These experimental results include the calculated experimental heat transfer rates of each specimen that have an expected trend based on changing fin length or base surface area. The corresponding theoretical heat transfer rates, however, are consistently different from the experimental values. As shown by the heat transfer rate error, each theoretical value is proportionally larger than the experimental value by a similar approximate percentage. This indicates the potential of a constant inconsistency in the calculation of experimental heat transfer rate that is changing the value by an order of magnitude. The trends between the theoretical and experimental values are significantly similar, so the previous analysis may be accurate, but the source of the consistent difference in magnitude must be found.

Conclusion

Many finned heat sink design factors have an impact on its ability to direct heat away from a heat-generating system as quantified by the rate of heat transfer it is able to provide. Through this experiment, we were able to determine that fin length has a positive impact on the heat transfer rate. Based on the insignificant change in heat flux, we know that the heat transfer rate per unit area does not change with increasing fin length, but the total heat transfer rate does increase due to the additional surface area provided by increased fin length. Increasing the base surface area of a finned surface decreases heat flux, although it does increase the total heat transfer rate due to the significant increase in overall surface area. Increasing base surface area and fin height are therefore both effective in increasing the heat transfer rate provided by a finned surface, and the results of this experiment demonstrate the quantity of this increase relative to the fin height and base surface area changes.

Recommendations for Improvement:
This experiment was limited in terms of the finned surface resources available and the factors we could evaluate. To improve this experiment, I recommend that additional materials should also be evaluated to determine how much material differences—particularly material properties such as the conduction and convection coefficient—affect change in the convection heat transfer rate given the same fin length and base surface area differences.

Further Investigation:
To expand on this experiment, I would suggest that the components of conduction and radiation are evaluated as well in calculation of the total heat flux and heat transfer rate for different types of finned surfaces. This experiment used a highly conductive material, anodized aluminum, with negligible resistance due to conduction and therefore minimal need to include this component in the total heat transfer rate calculation. It was also performed with a relatively low controlled hot plate temperature at which radiation is not a significant component. If this experiment were to be performed with a material of higher conduction resistance or a higher constant hot plate temperature to create a higher surface temperature, the conduction and/or radiation components could be evaluated in addition to the convection component of heat transfer.

Appendices

Appendix A: MATLAB Code

```
syms h
N = 48;
A_f = 76.68/(1000^2);
A_t = 4483.78/(1000^2);
t = 0.61 / 1000;
w = 3.30 / 1000;
P = 2*(t+w);
k = 1.12;
A_c = 2.01 / (1000^2);
L = 9.5 / 1000;
L_c = L + (t/2);
R = 14
eq = R == 1 / (h*A_t*(1-(N*A_f/A_t)*(1-(L_c*tanh((h*P/(k*A_c))^(1/2))/(L_c*(h*P/(k*A_c))^(1/2))))));
solve(eq, h)
```

Figure 5: MATLAB Code Used to Solve for Conduction Coefficient h—Specimen 1 Example

Appendix B: Time Spent on Experimental Process

Table 8

Activity	**Date(s)**
Designed experiment: Finned Surface Convection	11/4, 11/11
Determined research question	11/11
Found calculation methods	11/12 - 11/18
Created experimental procedure / setup	11/12 - 11/18
Received Specimens, gathered materials	11/18
Took experimental data	11/18
Performed data analysis	11/28 - 11/30
Wrote report	11/30 - 12/2

Appendix C: References

Carr. McMaster. (n.d.). Retrieved December 2, 2022, from https://www.mcmaster.com/heat-sinks/adhesive-mount-heat-sinks/overall-height~19-5-mm/

Carr. McMaster. (n.d.). Retrieved December 2, 2022, from https://www.mcmaster.com/heat-sinks/adhesive-mount-heat-sinks/overall-width~30mm/

Lee, J., Kim, Y., Jung, U., & Chung, W. (2013). Thermal conductivity of anodized aluminum oxide layer: The effect of electrolyte and temperature. *Materials Chemistry and Physics*, *141*(2-3), 680–685. https://doi.org/10.1016/j.matchemphys.2013.05.058